Abstracts for Workshop „New Trends in Geometric PDEs“

1.11. – 5.11.2021

Monday, 01.11.2021

Dejan Gajic (Radboud University)
Instabilities of extremal black holes
When Kerr black holes rotate at their maximally allowed angular velocity, they are said to be extremal. Extremal black holes display a variety of interesting phenomena that are not present in more slowly rotating black holes. I will introduce upcoming work on the existence of strong asymptotic instabilities of a non-axisymmetric nature for scalar waves propagating on extremal Kerr black hole backgrounds and I will discuss their connection with previous work on axisymmetric instabilities and the precise shape of late-time power law tails in the emitted radiation.

Claude Warnick (University of Cambridge)
Anti-de Sitter spacetimes
The anti-de Sitter (AdS) spacetimes, which solve Einstein's equations with a negative cosmological constant, are loosely speaking the Lorentzian analogue of hyperbolic geometries. In contrast to the situation with non-negative cosmological constant, the PDE problems which arise when studying the existence and stability of these spacetimes typically require a choice of boundary conditions on the (conformal) boundary of the spacetime. I will present an overview of progress in this area over the last few years, and discuss more recent work concerning black holes with exotic horizon topology.

Rita Teixeira da Costa (Princeton University)
Homogenization of the Einstein equations under symmetry
Due to their nonlinear nature, the Einstein equations are not closed under weak convergence: failure of compactness, due to oscillations and concentrations, produces an excess energy momentum tensor. In 1989, Burnett conjectured that, for vacuum sequences with high-frequency oscillations, the matter produced in this limit is captured by the Einstein-massless Vlasov model.

In this talk, we give a proof of Burnett's conjecture under some gauge and symmetry assumptions, improving previous work by Huneau—Luk from 2019. Our methods are more general, and apply to oscillating sequences of solutions to the wave maps equation in (1+2)-dimensions.

This is joint work with André Guerra (Institute for Advanced Study).

(remote) Jonathan Luk (Stanford University)
Singular solutions in general relativity
I will describe a few classes of solutions to the Einstein equations whose metrics are singular or highly oscillatory, and discuss the relation between them. These singular solutions include as special examples impulsive gravitational waves and null dust shell solutions. I will present some mathematical results concerning the initial value problem describing the propagation and interaction of impulsive gravitational waves, null dust shells, and other singular structures. This talk is based on works obtained with Cécile Huneau, Igor Rodnianski and Maxime Van de Moortel.

(remote) Yakov Shlapentokh-Rothman (University of Toronto)

Self-Similarity and Naked Singularities for the Einstein Vacuum Equations

We will start with an introduction to the problem of constructing naked singularities for the Einstein vacuum equations. Then we will discuss our previous work on the asymptotically self-similar regime for the Einstein equations and the corresponding connection to the "ambient metric" of Fefferman and Graham. Finally, we will explain our discovery of a fundamentally new type of self-similarity and show how this allows us to construct solutions corresponding to a naked singularity. This is all joint and ongoing work with Igor Rodnianski.

Tuesday, 02.11.2021

Cécile Huneau (CNRS & École Polytechnique)

Global existence for a system of quasilinear wave equations on a product space

In this talk, I will present a work with Annalaura Stingo (Davis--Polytechnique) where we study a system of quasilinear wave equations on the product of the Euclidean space with a one dimensional torus. This system is a toy model for Einstein equations with additional compact directions, present in Kaluza-Klein theory.

(remote) Marco Guaraco (Imperial College London)

Mean curvature flow in homology and foliations of hyperbolic 3-manifolds

We discuss global aspects of the mean curvature flow in closed 3-manifolds. One topic of interest is the long time behaviour of surfaces with non-vanishing mean curvature that are also homologically non-trivial. We show that such flows always become smooth after a finite time to then converge to a stable embedded minimal surface. Moreover, if the initial surface is incompressible and the ambient has negative curvature, then there exists a monotone isotopy between the initial and limit surfaces. Motivated by these results we then discuss mean-convex foliations and continuity properties of outermost minimal surfaces in quasi-Fuchsian hyperbolic 3-manifolds.

(remote) Grigorios Fournodavlos (Sorbonne Université)

Stable Big Bang formation

In this talk we will investigate the past dynamics of cosmological solutions to Einstein's equations, containing a Big Bang singularity. More precisely, we will focus on the classical generalised Kasner examples. The celebrated "singularity" theorem of Hawking tells us that the past of sufficiently small perturbations of such solutions are causally geodesically incomplete. However, it is not in general known whether such a degeneracy is related to the formation of a curvature singularity. In many cases, unstable dynamics are predicted, which add to the difficulty of the problem. We will discuss joint work with I. Rodnianski and J. Speck that classifies the behaviour of perturbed solutions in the so-called subcritical regime.

(remote) Lu Wang (California Institute of Technology)

Closed hypersurfaces of low entropy are isotopically trivial

We show that any closed connected hypersurface with low entropy is smoothly isotopic to the standard round sphere. This is joint work with Jacob Bernstein.

(remote) Peter Hintz (ETH Zürich)

Wave decay on asymptotically flat spacetimes

The focus of this talk is the study of asymptotic expansions of linear waves propagating on stationary and asymptotically flat spacetimes. On the spectral side, I will describe a hands-on approach to resolvent expansions near zero energy based on recent work by Vasy. I will discuss two main applications: first, sharp decay for scattering by short range potentials on $\mathbb{R}^3$; second, a strengthening of Price's Law on Kerr spacetimes.

Wednesday, 03.11.2021

(remote) Richard Bamler (University of California, Berkeley)

Compactness and partial regularity theory of Ricci flows in higher dimensions

We present a new compactness theory of Ricci flows. This theory states that any sequence of Ricci flows that is pointed in an appropriate sense, subsequentially converges to a synthetic flow. Under a natural non-collapsing condition, this limiting flow is smooth on the complement of a singular set of parabolic codimension at least 4. We furthermore obtain a stratification of the singular set with optimal dimensional bounds depending on the symmetries of the tangent flows. Our methods also imply the corresponding quantitative stratification result and the expected "L^p"-curvature bounds.

As an application we obtain a description of the singularity formation at the first singular time and a long-time characterization of immortal flows, which generalizes the thick-thin decomposition in dimension 3. We also obtain a backwards pseudolocality theorem and discuss several other applications.

(remote) Yoshikazu Giga (The University of Tokyo)

On a singular limit of a single-well Modica-Mortola functional and its applications

It is important to describe the motion of multi-phase boundaries. One typical way is to use the Kobayashi-Warren-Cater energy. It is a weighted total variation with single Modica-Mortola energy. The Modica-Mortola energy is very popular especially with double-well potential to handle two phase problems. It is important to characterize a singular limit of such a type of energies as the thickness parameter of a diffuse interface tends to zero. In the case of double-well potentials, such a problem is well studied and it is formulated, for example, as the Gamma limit under L^1 convergence.

However, if one considers the single-well Modica-Mortola functional, it turns out that L^1 convergence is too rough even in the one-dimensional problem.

We characterize the Gamma limit of a single-well Modica-Mortola functional under the topology which is finer than L^1 topology. In a one-dimensional case, we take the graph convergence. In higher-dimensional cases, it is more involved. As an application, we give an explicit representation of a singular limit of the Kobayashi-Warren-Carter energy. Since the higher-dimensional cases can be reduced to the one-dimensional case by a slicing argument, studying the one-dimensional case is very fundamental. A key idea to study the one-dimensional case is to introduce "an unfolding of a function" by changing an independent variable by the arc-length parameter of its graph. This is based on a joint work with Jun Okamoto (The University of Tokyo), Masaaki Uesaka (The University of Tokyo, Arithmer Inc.), and Koya Sakakibara (Okayama University of Science, RIKEN).

Tim Laux (Universität Bonn)

A new varifold solution concept for mean curvature flow: Convergence of the Allen-Cahn equation and weak-strong uniqueness

We propose a new weak solution concept for (two-phase) mean curvature flow which enjoys both (unconditional) existence and (weak-strong) uniqueness properties. These solutions are evolving varifolds, just as in Brakke's formulation, but are coupled to the phase volumes by a simple transport equation. First, we show that, in the exact same setup as in Ilmanen's proof [J. Differential Geom. 38, 417-461, (1993)], any limit point of solutions to the Allen-Cahn equation is a varifold solution in our sense. Second, we prove that any calibrated flow in the sense of Fischer et al. [arXiv:2003.05478] - and hence any classical solution to mean curvature flow - is unique in the class of our new varifold solutions. This is in sharp contrast to the case of Brakke flows, which a priori may disappear at any given time and are therefore fatally non-unique. Finally, we propose an extension of the solution concept to the multi-phase case which is at least guaranteed to satisfy a weak-strong uniqueness principle. This is joint work with Sebastian Hensel (U Bonn).

Thursday, 04.11.2021

(remote) Elena Mäder-Baumdicker (TU Darmstadt)

Geometry of complete minimal surfaces and the Willmore Morse index of their inversion

A complete minimal surfaces with embedded planar ends can be compactified via inversions at spheres. The resulting surfaces are Willmore surfaces in Euclidean space. We explain how the Willmore Morse index of these surfaces is related to unbounded Area-Jacobi fields on the corresponding minimal surface. Also several geometric properties of the minimal surfaces can be used to get results about the Willmore Morse index of the inverted surface. An example is whether the asymptotic planes at the ends meet at one point. This talk is based on joint work with Jonas Hirsch and Rob Kusner.

(remote)Tobias Lamm (Karlsruher Institut für Technologie)

Diffusive stability results for the harmonic map flow and related equations

The goal of this talk is to introduce the audience to the theory of diffusive stability in the context of the harmonic map flow. This theory is useful when studying stability results for parabolic equations and we will illustrate its use for geometric equations such as the harmonic map flow. Additionally, we use this theory in order improve various uniqueness results for solutions with rough initial data.

Eleonora di Nezza (École Polytechnique)

Entropy & Energy

One of the approaches to find Kähler-Einstein metrics is the variational approach which consists in minimising a functional defined on the space of Kähler metrics. Probability measures with finite entropy plays a central role since "finite entropy" implies "finite energy”.

In this talk we show that finite entropy Monge-Ampère measures belong to a higher finite energy class and we provide the optimal exponent.

This is a joint work with Chinh Lu and Vincent Guedj.

Julian Fischer (Institute of Science and Technology Austria)
Uniqueness and stability properties of weak solutions to multiphase mean curvature flow: An approach based on the local structure of the energy landscape
In mean curvature flow (MCF), topological changes occur naturally, a basic example being the shrinkage and disappearance of a phase. As a consequence, classical solution concepts for mean curvature flow are in general limited to short-time existence theorems. At the same time, the transition from strong to weak solution concepts for PDEs is prone to incurring "unphysical" non-uniqueness of solutions, as for instance the concept of Brakke solutions for MCF demonstrates. For two-phase mean curvature flow, the concept of viscosity solutions by Chen-Giga-Goto and Evans-Spruck nevertheless provides a notion of weak solutions with basically optimal uniqueness properties.

On the other hand, multiphase mean curvature flow does not admit a comparison principle; thus, the concept of viscosity solutions is not applicable. In fact, no solution concept had been available that ensures global existence of solutions and at the same time guarantees uniqueness prior to the first topology change. By introducing a novel concept of "gradient flow calibrations", we establish a weak-strong uniqueness principle for BV solutions to planar multiphase mean curvature flow: For sufficiently regular initial data, weak (BV) solutions to planar multiphase mean curvature flow are unique prior to the first topological change. As the uniqueness of evolutions may fail past certain topology changes, this result is basically optimal.

In the last part of the talk, we discuss further applications of our new concept of "gradient flow calibrations", including the quantitative convergence of diffuse-interface (Allen-Cahn) approximations for mean curvature flow, as long as a strong solution to the latter exists.

(remote) André Neves (University of Chicago)
Minimal surfaces in hyperbolic manifolds
The study of geodesics in negatively curved manifolds is a rich subject which has been at the core of geometry and dynamical systems. Comparatively, much less is known about minimal surfaces on those spaces. I will survey some of the recent progress in that area.

Friday, 05.11.2021

(remote) Yoshihiro Tonegawa (Tokyo Institute of Technology)
Existence of canonical multi-phase mean curvature flows
I present a recent existence result for multi-phase mean curvature flow starting from arbitrary closed set of locally finite co-dimension one Hausdorff measure. The flow is expected to go through various topological changes and is defined in the framework of geometric measure theory. This is a joint work with Salvatore Stuvard (Univ of Milan).

(remote) Matteo Novaga (Università di Pisa)
Anisotropic and fractional mean curvature flow of mean convex sets
I will introduce the evolution of sets by anisotropic or fractional mean curvature, and show the preservation along the flow of two geometric properties: the mean convexity and the outward minimality. I will discuss the convergence of the time-integrated perimeters of the discrete evolutions to the perimeter of the limit flow, and the uniqueness of the flat flow obtained in the limit. The main tools are the level set formulation and the minimizing movement scheme.

(remote) Hung Vinh Tran (University of Winsconsin Madison)

Level-set forced mean curvature flow with the Neumann boundary condition

We study a level-set forced mean curvature flow with the homogeneous Neumann boundary condition. We first show that the solution is Lipschitz in time and locally Lipschitz in space. Then, under an additional condition on the forcing term, we prove that the solution is globally Lipschitz. We obtain the large time behaviour of the solution in this setting and study the large time profile in the radially symmetric setting. Finally, we give two examples demonstrating that the additional condition on the forcing term is sharp, and without it, the solution might not be globally Lipschitz. Joint work with Jang, Kwon, Mitake.

(remote) Susanna Terracini (Università degli Studi di Torino)

Free boundaries in segregation problems