Next goal: Define an L-theory space $L(\mathcal{C},Ϙ)$ and describe the higher L-groups

Observation: $(X,q) \sim (X',q') \iff \exists$ diagram of the shape

$$\begin{tikzcd} & W \arrow[dl, "f"'] \arrow[dr, "f'"] & \\ X & & X' \end{tikzcd} \quad + \quad \eta : f^*(q) \sim f'^*(q') \in \Omega^\infty Ϙ(W)$$

$$\begin{tikzcd} & W \arrow[dl] \arrow[dr] \arrow[dd] & \\ X \arrow[d, phantom, "\wr"] & & X' \arrow[d, phantom, "\wr"] \\ D_Ϙ X \arrow[r] & D_Ϙ W & D_Ϙ X' \arrow[l] \end{tikzcd}$$

s.th. a non-degeneracy condition is satisfied

Definition: Let $(\mathcal{C},Ϙ)$ be a Poincaré ∞-category. Define

$$\mathrm{Cat}_\infty^p \ni \mathcal{Q}_n(\mathcal{C},Ϙ) = \left[\mathrm{Fun}(\mathcal{T}_n^{op}, \mathcal{C}),\ \mathrm{Fun}(\mathcal{T}_n^{op},\mathcal{C})^{op} \simeq \mathrm{Fun}(\mathcal{T}_n, \mathcal{C}^{op}) \xrightarrow{Ϙ_*} \mathrm{Fun}(\mathcal{T}_n, Sp) \xrightarrow{\lim} Sp \right]$$

$\mathcal{T}_n$ = poset of non-empty faces of Δ^n · $Q_1(\mathcal{C},Ϙ)$ $(\mathcal{T}_1 = \bullet \to \bullet \leftarrow \bullet)$

e.g. $\mathcal{Q}_0(\mathcal{C},Ϙ) = (\mathcal{C},Ϙ)$ and $\mathcal{Q}_1(\mathcal{C},Ϙ)$ has objects: ⊛ $\begin{tikzcd} & W \arrow[dl] \arrow[dr] & \\ X & & X' \end{tikzcd}$ + $\begin{tikzcd} Ϙ_1(*) \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & Ϙ(X') \arrow[d] \\ Ϙ(X) \arrow[r] & Ϙ(W) \end{tikzcd}$

Yoneda's lem. $\to$ ‖

Fact: Poincaré objects of $\mathcal{Q}_1(\mathcal{C},Ϙ)$ $\hat{=}$ Lagrangian in $(X \oplus X', q \oplus -q')$ =: cobordisms between (X,q) and (X',q').

Proposition: there is a functor $\Delta^{op} \times \mathrm{Cat}_\infty^p \longrightarrow \mathrm{Cat}_\infty^p$ sending $([n],(\mathcal{C},Ϙ))$ to $\mathcal{Q}_n(\mathcal{C},Ϙ)$.

In addition, all simpl. face maps $\mathcal{Q}_n(\mathcal{C},Ϙ) \xrightarrow{d_i} \mathcal{Q}_{n-1}(\mathcal{C},Ϙ)$ are split Poincaré-Verdier projections.

Definition: $L : \mathrm{Cat}_\infty^p \longrightarrow Spc$ is the geometric realisation of

$$[n] \longmapsto \mathrm{Pn}(\mathcal{Q}_n(\mathcal{C},Ϙ))$$

$\mathrm{Pn}(-) = \mathrm{Map}_{\mathrm{Cat}_\infty^p}((Sp^{fin}, Ϙ^u), -)$ + Cat_∞^p is pre additive.

$\Downarrow$

$\mathrm{Pn}(\mathcal{C},Ϙ)$ is an $\mathbb{E}_\infty$-space for all $(\mathcal{C},Ϙ)$ $\rightsquigarrow$ $L(\mathcal{C},Ϙ)$ is also an $\mathbb{E}_\infty$-space

Exercise 9: Show that $\pi_0 L(\mathcal{C},Ϙ) \cong L_0(\mathcal{C},Ϙ)$.
Deduce that $L(\mathcal{C},Ϙ)$ is a grouplike $\mathbb{E}_\infty$-space and hence a connective spectrum

Exercise 10: Show that $\left| [n] \mapsto \mathrm{Fm}(\mathcal{Q}_n(\mathcal{C},Ϙ)) \right|$ is contractible.
Hint: use extension by 0's to define an extra degeneracy.

Definition: Let $\mathrm{Met}(\mathcal{C},Ϙ)$ be the Poincaré subcategory of $\mathcal{Q}_1(\mathcal{C},Ϙ)$ on objects of the form $\begin{tikzcd} & W \arrow[dl] \arrow[dr] & \\ 0 & & X \end{tikzcd}$

Concretely: $Ϙ^{met}(L \to X) = \mathrm{fib}(Ϙ(X) \to Ϙ(L))$ + $D_{Ϙ^{met}}(L \to X) = (D_Ϙ(L/X) \to D_Ϙ X)$

earlier analysis $\Rightarrow$ $\mathrm{Pn}(\mathrm{Met}(\mathcal{C},Ϙ)) = \{\text{Lagrangians } (L \to X, \eta)\}$ X Poincaré objects of $\mathcal{C},Ϙ$

Prop: $(\mathcal{C}, \OmegaϘ) \xrightarrow{i} \mathrm{Met}(\mathcal{C},Ϙ) \xrightarrow{t} (\mathcal{C},Ϙ)$ is a split Poincaré-Verdier sequence.

$$(L \to X) \longmapsto X$$
$$y \longmapsto (y \to 0)$$

i.e. fibre + cofibre sequence in Cat_∞^p, s.th. t (just as functor of stable ∞-cats) admits left + right adjoints.

proof: · $Ϙ^{met}(i(y)) = Ϙ^{met}(y \to 0) = \mathrm{fib}(\underset{\substack{\| \\ 0}}{Ϙ(0)} \to Ϙ(y)) = \OmegaϘ(y)$ ✓

· $Ϙ^{met}(L \to X) = \mathrm{fib}(Ϙ(X) \to Ϙ(L)) \longrightarrow Ϙ(X)$

defines the datum of herm. functor

$t(D_{Ϙ^{met}}(L \to X)) \simeq D_Ϙ(X)$

the adjoint need not be Poincaré functors.

· split PV proj. is implied by: t has fully faithful left adjoint ℓ and $Ϙ \simeq Ϙ^{met} \circ \ell$

$Ϙ$ = left Kan ext. from $Ϙ^{met}$

here: $\ell(X) = (0 \to X)$ ✓ $Ϙ^{met}(\ell(X)) = \mathrm{fib}(Ϙ(X) \to Ϙ(0)) = Ϙ(X)$. ∎

(Ranicki, Lurie)

Theorem: $L : \mathrm{Cat}^p_\infty \to \mathcal{S}p_{\geq 0}$ sends (split) Poincaré-Verdier sequences to fibre sequences

Proposition: $\pi_n L(\mathcal{C}, \varphi) \cong L_0(\mathcal{C}, \Omega^n \varphi)$

proof: Induction over n. $n = 0$ ✓ (Exercise 9)

$n \geq 1$:

$$\pi_n L(\mathrm{Met}(\mathcal{C},\varphi)) \to \pi_n(L(\mathcal{C},\varphi)) \to \underset{\substack{\text{induct.}\ \cong\\ L_0(\mathcal{C}, \Omega^{n-1}\Omega\varphi)}}{\pi_{n-1}(L(\mathcal{C},\Omega\varphi))} \to \pi_{n-1} L(\mathrm{Met}(\mathcal{C},\varphi))$$

on underlying categories $\mathrm{Fun}(\mathcal{J}_n^{op}, \mathrm{Fun}(\Delta^1, \mathcal{C})) \simeq \mathrm{Fun}(\Delta^1, \mathrm{Fun}(\mathcal{J}_n^{op}, \mathcal{C}))$

suffices thus to prove $L(\mathrm{Met}(\mathcal{C},\varphi)) \simeq *$.

$$L(\mathrm{Met}(\mathcal{C},\varphi)) = \left| [n] \mapsto \mathrm{Pn}(\rho_n(\mathrm{Met}(\mathcal{C},\varphi))) \right|$$

Observation: $\rho_n \mathrm{Met}(\mathcal{C},\varphi) \simeq \mathrm{Met}(\rho_n(\mathcal{C},\varphi))$

obj. $(\mathcal{C},\varphi)$ — red. 2 excisive

morph. $F : \mathcal{C} \to \mathcal{C}'$ + $\eta : \varphi \to \varphi' \circ F^{op}$

$n = 1$

$$\begin{tikzcd} X & W \arrow[l] \arrow[r] & X' & \rho_1 \\ Y \arrow[from=u] & V \arrow[l] \arrow[r] \arrow[from=u] & Y' \arrow[from=u] & \rho_1 \end{tikzcd} \quad \Delta^1$$

$(\varphi_n)^{met}$ = fibre of pullbacks of φ's
$\|$
$(\varphi^{met})_n$ = pullback of fibre of φ's.

Proposition: (Ranicki) $\mathrm{Pn}(\mathrm{Met}(\mathcal{C},\varphi)) \overset{\text{alg. Thom isomorphism}}{\simeq} \mathrm{Fm}(\mathcal{C}, \Omega\varphi)$

reason: the forgetful functor $\mathrm{Cat}^p_\infty \xrightarrow{u} \mathrm{Cat}^h_\infty$ has left + right adjoints ℓ, r,

+ $r(u(\mathcal{C},\varphi)) = \mathrm{Met}(\mathcal{C},\Sigma\varphi)$.

Hence:
$$\mathrm{Pn}(\mathrm{Met}(\mathcal{C},\varphi)) = \mathrm{Map}_{\mathrm{Cat}^p_\infty}((\mathcal{S}p^f, \varphi^u), \mathrm{Met}(\mathcal{C},\varphi)) = \mathrm{Map}_{\mathrm{Cat}^p_\infty}((\mathcal{S}p^f, \varphi^u), r\,u(\mathcal{C},\Omega\varphi))$$
$$= \mathrm{Map}_{\mathrm{Cat}^h_\infty}(u(\mathcal{S}p^f, \varphi^u), u(\mathcal{C},\Omega\varphi)) = \mathrm{Fm}(\mathcal{C},\Omega\varphi).$$

the equivalence is implemented by an alg. surgery as we will see tomorrow. ▪ Prop.

Thus:
$$L(\mathrm{Met}(\mathcal{C},\varphi)) = \left| [n] \mapsto \mathrm{Pn}(\rho_n \mathrm{Met}(\mathcal{C},\varphi)) \right|$$
$$= \left| [n] \mapsto \mathrm{Pn}(\mathrm{Met}(\rho_n(\mathcal{C},\varphi))) \right| = \left| [n] \mapsto \mathrm{Fm}(\rho_n(\mathcal{C},\Omega\varphi)) \right| \overset{\text{Exercise 10}}{=} * \quad ▪$$

- Thus, $\pi_n L(\mathcal{C},\varphi)$ has a uniform generators and relations description (Poincaré objects modulo null cobordant ones)

Construction: Let R be a ring, M an invertible module w/ involution over M.

Define: $W^s(R;M) \longrightarrow L_0(\mathcal{D}^p(R); \varphi^{gs}_M) \longrightarrow L_0(\mathcal{D}^p(R); \varphi^s_M)$

by: $[P,b] \mapsto [P[0], b \in \Omega^\infty \varphi^{gs}_M(P)]$ using $\Omega^\infty \varphi^{gs}_M(P) = \mathrm{Hom}_{R\otimes R}(P \otimes P, M)^{C_2}$

$[P,b,q] \longmapsto [P[0], (b,q) \in \pi_0 \Omega^\infty \varphi^q_M(P)]$

Likewise: $W^q(R;M) \longrightarrow L_0(\mathcal{D}^p(R); \varphi^{gq}_M)$ using $\pi_0 \Omega^\infty \varphi^{gq}_M(P) \simeq \Omega^\infty \varphi^{gq}_M(P) = \mathrm{Hom}_{R\otimes R}(P\otimes P, M)_{C_2}$.

$$\begin{tikzcd} W^q(R;M) \arrow[r] \arrow[dr, dashed] & L_0(\mathcal{D}^p(R); \varphi^{gq}_M) \\ & L_0(\mathcal{D}^p(R); \varphi^q_M) \arrow[u] \end{tikzcd}$$

Keep in mind the question: What is the relation between $W^s(\mathbb{R})$ and $L_0(\mathcal{D}^p(\mathbb{R}), Q^{gs}_M)$, Q^{s}_M ...

Algebraic Surgery

Idea: simplify representatives of $L_0(\mathcal{C}, Q)$ without changing their cobordism class.

Definition: A surgery datum on a form (X, q) consists of a pair (S, η) where

- $S \xrightarrow{f} X$ is a map, and
- $\eta : f^*(q) \sim 0$ in $\Omega^\infty Q(S)$

$\iff$ point of $\mathrm{Fm}(\mathrm{Met}(\mathcal{C}, Q))$
$$\begin{tikzcd} \mathrm{Fm}(\mathrm{Met}(\mathcal{C},Q)) \arrow[d, "t"] \\ \mathrm{Fm}(\mathcal{C},Q) \end{tikzcd}$$
(X, q)

Proposition: there is a Poincaré functor $\chi : \mathrm{Met}(\mathcal{C}, Q) \longrightarrow \mathcal{P}_1(\mathcal{C}, Q)$ sending a surgery datum to the trace of its surgery inducing an equivalence
$$\underbrace{\mathrm{Fm}(\mathrm{Met}(\mathcal{C},Q)) \times_{\mathrm{Fm}(\mathcal{C},Q)} \mathrm{Pn}(\mathcal{C},Q)}_{\text{one Poincaré object w/ a surgery datum}} \longrightarrow \underbrace{\mathrm{Pn}(\mathcal{P}_1(\mathcal{C},Q))}_{\text{Cobordisms between Poincaré objects}}$$

<u>sketch</u>: the span associated to $\chi((S \xrightarrow{f} X, \eta))$ is constructed as follows:

$$\begin{tikzcd} S \arrow[r, equal] \arrow[d] & S \arrow[r] \arrow[d, "f"] \arrow[dr, phantom, "\eta"] & 0 \arrow[d] \\ X_f \arrow[r, "\alpha"] \arrow[d, "\beta"] & X \arrow[r] \arrow[d] & D_Q S \arrow[d, equal] \\ X_S \arrow[r] & X/S \arrow[r] & D_Q S \end{tikzcd}$$

$X \simeq D_Q X \to D_Q S$, $D_Q f$

on forms
$$\begin{tikzcd} & Q(X_S) \arrow[d] & \\ Q^{met}(S \to X_f) \arrow[r] \arrow[d] & Q(X_f) \arrow[r] \arrow[d] & Q(S) \arrow[d, "can"] \\ \mathrm{map}(X_S, DS) \arrow[r] & \mathrm{map}(X_f, DS) \arrow[r] & \mathrm{map}(S, DS) \end{tikzcd}$$

$q_S \in Q(X_S)$, $q_S \mapsto \bar{q}_S$; $0 \mapsto \alpha^*(q)$; $\alpha^*(q) \xmapsto{\eta} 0$; $0 \mapsto 0 = (X_S \to X/S \to DS)$

horizontal + vertical fibre sequences

$$\begin{tikzcd} Q(X_f) \arrow[r] & \mathrm{map}(X_f, DX_f) \arrow[d] \\ & \mathrm{map}(X_f, DS) \end{tikzcd}$$

This is the effect of the functor χ on forms. We indicate that it restricts to an equivalence
$$\underbrace{\mathrm{Fm}(\mathrm{Met}(\mathcal{C},Q)) \times_{\mathrm{Fm}(\mathcal{C},Q)} \{0\}}_{= \mathrm{Fm}(\mathcal{C}, \Omega Q) \text{ surgery datum}} \overset{\text{alg. Thom isom.}}{\simeq} \underbrace{\mathrm{Pn}(\mathcal{P}_1(\mathcal{C},Q) \times_{(\mathcal{C},Q)} \{0\})}_{\mathrm{Pn}(\mathrm{Met}(\mathcal{C},Q))}$$

$(S, q \in \Omega Q(S)) \longmapsto (S \to 0, q) \longmapsto$ output of surgery on 0

consider the map
$$\begin{tikzcd} N \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & 0 \arrow[d] \\ L \arrow[r] & X \end{tikzcd}$$
$\longleftrightarrow (L \to X, \eta \in Q^{met}(L \to X))$

$Q^{met}(L \to X) \to Q^{met}(N \to 0) \simeq \Omega Q(N)$

$\eta \longmapsto q$

Exercise 11: Show that the underlying object of the surgery output of $(S \to 0, \eta \in \Omega Q(S))$ is given by $\mathrm{cofib}(S \xrightarrow{\eta_\#} \Omega D_Q S)$.

$$\begin{tikzcd} & S \arrow[r, equal] \arrow[d] & S \arrow[r] \arrow[d] & 0 \arrow[d] \\ L = & X_0 \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & D_Q S \arrow[d, equal] \\ X = & X_S \arrow[r] & \Sigma S \arrow[r] & D_Q S \end{tikzcd}$$

■

Next goal: Show that the map $W^s(R;M) \longrightarrow L_0(\mathcal{D}^p(R); Ϙ_M^{gs})$ is an isomorphism.

in particular, need to argue that, up to cobordism, Poincaré objects for $Ϙ_M^{gs}$ can be assumed to be of the form $P[0]$ for $P \in \mathrm{Proj}(R)$.

Lemma: Let R, M as before. Let $X \in \mathcal{D}^p(R)$. TFAE

X m conn. "$\pi_i(X)$ / $H_i(X) = 0$ if $i < m$"

1) X is m-connective and $D_M X$ is $(-n)$-connective

$P_i \in \mathrm{Proj}(R)$ sitting in hom. degree i

2) X is repr. by a chain complex $0 \to P_n \to P_{n-1} \to \dots \to P_{m+1} \to P_m \to 0$

proof: 2) ⇒ 1) step 1: X repr. by some finite chain complex C, ie. C_i is f.g. proj. and almost all $C_i = 0$

step 2: Since X is m-conn. ⇒ achieve $C_i = 0$ for $i < m$

step 3: DX repr. by applying $\mathrm{Hom}_R(-, M)$ to the representing chain complex C

step 4: since DX is $(-n)$ connective ⇒ achieve $\mathrm{Hom}_R(C, M)_i = 0$ for $i < -n$

step 5: $X \simeq D^2 X$ is now of the claimed shape. ∎

e.g. $Ϙ_M^{gs}, Ϙ_M^{ge}, Ϙ_{\mathbb{Z}}^{b}$

Definition: R, M as always. $Ϙ$ M-compatible + assume $Ϙ(P)$ is an EM spectrum in degree 0 for any $P \in \mathrm{Proj}(R)$. Let $W(\mathrm{Proj}(R); Ϙ) = \left(\left\{ \begin{array}{l} \text{isom. classes of unim.} \\ Ϙ\text{-forms on f.g. proj} \end{array} \right\}, \oplus \right) \Big/ \langle \text{metabolic forms} \rangle$ sometimes called "strictly" metabolic

Proposition: Under the above assumptions, the canonical map

$W(\mathrm{Proj}(R); Ϙ_0) \longrightarrow L_0(\mathcal{D}^p(R), Ϙ)$ is an isomorphism.

proof: surjectivity: Let (X, q) be a Poincaré object for $Ϙ$.

if X is connective ⇒ $X \overset{q_\#}{\simeq} DX$ is also connective.

hence $X \simeq P[0]$ by previous lemma. ✓

in general, X is $(-k)$ connective for some $k \geq 0$ (or $X = 0$) assume $-k = \min_i \{ \pi_i(X) \neq 0 \} < 0$

By the lemma ⇒ $\pi_{-k}(X) \overset{\exists}{\twoheadleftarrow} P$ f.g. proj. ⇝ $P[-k] \xrightarrow{f} X$ in $\mathcal{D}^p(R)$

now: $Ϙ(P[-k])$ is k-connective ⇒ $\pi_0 Ϙ(P[-k]) = 0$.

⇒ $f^*(q) \sim 0$ in $\Omega^\infty Ϙ(P[-k])$ so any nullhtpy η makes $(P[-k] \xrightarrow{f} X, \eta)$ a surgery datum.

Exercise 12: the surgery output (X', q') satisfies that X' is $(-k+1)$-connective.

Injectivity: Assume given $P \in \mathrm{Proj}(R)$ and $q \in Ϙ(R)$, and assume that its image in $L_0(\mathcal{D}^p(R), Ϙ)$ vanishes, i.e. $\exists\, L \in \mathcal{D}^p(R)$, a map $f: L \to P[0]$ and a nullhtpy $\eta: f^*(q) \sim 0$

such that $L \to P[0] \overset{q_\#}{\simeq} DP[0] \to DL$ is a fibre sequence.

abstractly, $(L \to P, \eta) \in \mathrm{Pn}(\mathrm{Met}(\mathcal{D}^p(R), Ϙ))$ lifting $(P, q) \in \mathrm{Pn}(\mathcal{D}^p(R), Ϙ)$

Exercise 12: then L is represent. by a complex $0 \to Q \to N \to 0$, $\mathrm{degree}(N) = -1$.

Claim: Using surgeries, we can make L (-1)-connective.

proof: assume $-k = \min_i \{\pi_i(L) \neq 0\}$. Pick a surj. $Q[-k] \to \pi_{-k}(L)$. and the ass. map $Q[-k] \to L$. The diagram

$$\begin{array}{ccc} Q[-k] & \longrightarrow & 0 \\ \downarrow & & \downarrow \\ L & \longrightarrow & P \end{array}$$

commutes as $\mathrm{map}_R(Q[-k], P) = \mathrm{Ext}^k_R(Q, P) = 0$.

$+\ Ϙ^{met}(Q[-k] \to 0) = \Omega Ϙ^{met}(Q[-k])$ is $(k-1)$-connective.

Exercise 13: surgery output ass. to the map $\left(\begin{smallmatrix} Q[-k] \\ \downarrow \\ 0 \end{smallmatrix} \to \begin{smallmatrix} L \\ \downarrow \\ P \end{smallmatrix}\right)$ in $\mathrm{Met}(\mathcal{D}^p(R))$

is of the form $L' \to P$ with L $(-k+1)$-connective

Inductively $\rightsquigarrow$ L can be assumed (-1)-connective. Writing L as $(0 \to Q \to N \to 0)$ [$(*)$: $0 \to N$ — Ex. 12]

we find a map $N[-1] \xrightarrow{(*)} L$ whose cofibre is $Q[0]$.

Consider the map

$$\begin{array}{ccc} N[-1] & \longrightarrow & L \\ \| & & \downarrow \\ N[-1] & \longrightarrow & P \end{array}$$

in $\mathrm{Met}(\mathcal{D}^p(R))$. $Ϙ^{met}(N[-1] = N[-1]) = 0$

$\rightsquigarrow$ can do surgery. Output is a Lagrangian of the form $Q[0] \longrightarrow P \oplus (N \oplus DN)$

where the form on $P \oplus (N \oplus DN)$ is (q, q') with q' metabolic ($DN \to N \oplus DN$ is a Lagrangian for q')

$\Rightarrow$ in $W(\mathrm{Proj}(R); Ϙ)$ have: $[P, q] = [P, q] + [N \oplus DN, q'] = [P \oplus (N \oplus DN), q \oplus q'] = 0$ since $Q[0]$ is a Lagrangian here. $\blacksquare$

Corollary: $L_0(\mathcal{D}^p(R), Ϙ^{gs}) \cong W^s(R; M)$ and $L_0(\mathcal{D}^p(R), Ϙ^{gq}) \cong W^q(R; M)$

Definition: Let $Ϙ: \mathcal{D}^p(R)^{op} \to \mathrm{Sp}$ be an M-compatible Poincaré structure, associated to

a map $N \xrightarrow{\alpha} M^{tC_2}$ of R-modules. We say that $Ϙ$ is

1) m-quadratic if N is m-connective — $\pi_i(N) = 0$ for $i < m$

2) r-symmetric if the fibre of α is $(-r)$-truncated. — $\pi_i(\mathrm{fib}(\alpha)) = 0$ for $i > -r$

Exercise 14: 1) φ is m-quadratic $\Leftrightarrow$ $L_\varphi(X)$ is $(m+k)$-connective whenever DX is k-connective

2) φ is r-symmetric $\Leftrightarrow$ $\mathrm{fib}(\varphi(X) \to \varphi^s(X))$ is $(-r-k)$-truncated whenever X is k-connective.

Next goal: make qualitative statements about the maps

$$L(\mathcal{D}^p(R), \varphi^q_M) \longrightarrow L(\mathcal{D}^p(R), \varphi) \longrightarrow L(\mathcal{D}^p(R), \varphi^s_M)$$

for an M-compatible Poincaré structure φ

Recall: we calculated $L_0(\mathcal{D}^p(R), \varphi)$ in terms of classical forms if $\varphi(P)$ = EM-spectrum in degree 0

Thm: Let φ be an m-quadr. M-comp. Poincaré structure on $\mathcal{D}^p(R)$. Then

$$\pi_n L(\mathcal{D}^p(R), \varphi^q_M) \longrightarrow \pi_n L(\mathcal{D}^p(R), \varphi) \text{ is } \begin{cases} \text{an isom. for } & n \leq 2m-3 \\ \text{surjective for } & n \leq 2m-2 \end{cases}$$