Noncommutative geometry meets topological recursion
August 16th-20th 2021, WWU Münster

General information

• All given times in the schedule are Germany time (CEST). For instance:
- California = -9h
- Colorado/Saskatchewan = -8h
- Kansas/Texas = -7h
- US East Coast/Ontario = -6h
- UK = -1h
- Israel = +1h
- Chennai = + 3h30
- China = +6h
- Pohang = +7h

Note that Central European Summer Time (CEST) is 2 hours ahead of Coordinated Universal Time (UTC), e.g. 9:00 CEST = 7:00 UTC.

• Videos of the talks are posted (when speakers agree) on the Youtube channel: `https://www.youtube.com/c/mathematicsmunster`

Monday, August 16th

9am — *Elba Garcia–Failde*
Introduction to topological recursion 1

10am — Tea & coffee

10.30am — *Walter van Suijlekom*
Non-commutative geometry and spectral triples 1

11.30am — Break

Noon — *Sergey Shadrin*
Arnold's trinity of algebraic $2d$ gravitation theories

1pm — Lunch break

3pm — *Roland Speicher*
Free probability theory 1

4pm — Tea & coffee

4.30pm — *Gong talks*

5.30pm — Short break

5.45pm — *Gong talks*

7pm — Reception

Tuesday, August 17th

9am — *Elba Garcia–Failde*
Introduction to topological recursion 2

10am — Tea & coffee

10.30am — *Walter van Suijlekom*
Non-commutative geometry and spectral triples 2

11.30am — Break

Noon — *Roberta Iseppi*
The BV-BRST cohomology for U(n)-gauge theories
induced by finite spectral triples

1pm — Lunch break

3pm — *Bertrand Eynard*
(Mixed) topological recursion and the two-matrix model 1

4pm — Tea & coffee

4.30pm — *Lisa Glaser*
A picture of a spectral triple

6pm — Conference dinner

Wednesday, August 18th

9am — *Elba Garcia–Failde*
Introduction to topological recursion 3

10am — Tea & coffee

10.30am — *Roland Speicher*
Free probability theory 2

11.30am — Break

Noon — *Camille Male*
Freeness over the diagonal and the global fluctuations of Wigner matrices

1pm — Lunch break

3pm — *Bertrand Eynard*
(Mixed) topological recursion and the two-matrix model 2

4pm — Tea & coffee

4.30pm — *Yan Soibelman*
Wall-crossing structures and exponential integrals

Thursday, June 17th

9am — *Walter van Suijlekom*
Non-commutative geometry and spectral triples 3

10am — Tea & coffee

10.30am — *Roland Speicher*
Free probability theory 3

11.30am — Break

Noon — *Kurusch Ebrahimi-Fard*
An operadic derivation of the twisted factorisation for the operator-valued T-transform

1pm — Lunch break

3pm — *Bertrand Eynard*
(Mixed) topological recursion and the two-matrix model 3

4pm — Tea & coffee

4.30pm — *James Mingo*
The infinitesimal Weingarten calculus

6pm — Barbecue

Friday, June 18th

9am — *Piotr Śniady*
Representation theory from the random matrix perspective

10am — Tea & coffee

10.30am — *Séverin Charbonnier*
Topological recursion for fully simple maps

11.30am — Break

Noon — *Alexander Hock*
From noncommutative QFT to blobbed topological recursion

1pm — Lunch break, end of the workshop

Mini-courses (3h each)

• **Bertrand Eynard** (IPhT CEA Saclay)
(Mixed) topological recursion and the two-matrix model

An issue in non-commutative probabilities, is to compute "mixed traces" of products of random matrices: typically expectation values of the type

$$\mathbb{E}\left(\operatorname{Tr} A^{k_1} B^{l_1} A^{k_2} B^{l_2} ...\right)$$

with a joint random matrix measure $P(A, B)$, which we assume to be invariant under simultaneous unitary conjugation of A and B. The prototype is the 2-matrix model with $A, B \in H_N \times H_N$:

$$P(A, B) = e^{-\operatorname{Tr} V(A)} e^{-\operatorname{Tr} \tilde{V}(B)} e^{\operatorname{Tr} AB} \mathrm{d}A\mathrm{d}B.$$

Another phrasing of this question consists in diagonalizing $A = UXU^\dagger$ and $B = VYU^\dagger$, and calling $W = U^{-1}V$. At fixed spectrum (fixed X and Y), mixed traces amount to computing expectation values of polynomials of W with the Harish-Chandra measure:

$$\int_{W\in G} dW e^{\operatorname{Tr} XWYW^{-1}} W_{i_1,j_1} W^\dagger_{i_2,j_2} W_{i_3,j_3} W^\dagger_{i_4,j_4} \cdots$$

where dW is the Haar measure on the group $G = U(N)$ (can be generalized to other compact Lie groups).

This problem is also closely related to a problem of enumeration of maps in combinatorics: maps with faces of 2 colors, called the "Ising model on random maps". Mixed traces correspond to counting maps with boundaries and with a given sequence of colors along the boundaries. In comparison, non-mixed traces (traces of powers of only A or only B) would count maps with uniformly colored boundaries.

For the 2 matrix model, all these expectation values have been computed, the results involve determinants of some "universal matrices", and yields surprisingly simple answers.

Moreover recursion formulas for their large N expansion are known, and are an extension of the "topological recursion". In the map interpretation, the large N expansion amounts to counting maps of higher topologies, the power of N being the Euler characteristics of the maps counted. The large N leading order amounts to counting planar maps.

In this series of lecture we will introduce the 2-matrix model and the issue of mixed traces, then we shall give the answers as formulas. Some formulas will be proved during the lectures, but the main goal is to explain how to use the formulas for practical computations.

If times permits, the formulas for all compact Lie groups will be addressed.

We shall largely follow the chapter 8 of the book *Counting surfaces*, B. Eynard, Birkhäuser 2016.

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• **Elba Garcia-Failde** (IRIF, Paris 7)
Introduction to topological recursion

In this mini-course I will introduce the universal procedure of topological recursion, both by treating examples and by presenting the general formalism. We will study

the classical case of the Hermitian matrix model in detail, which combinatorially corresponds to ribbon graphs, beginning from the loop equations, which correspond to Tutte's recursion in the combinatorial setting. This will be the starting point to make the connection to free probability, which moreover provides a combinatorial way of exploring the variation of the topological recursion output when applying a symplectic transformation to the input. Apart from the (conjectural) property of symplectic invariance, topological recursion has many other interesting features and, together with its generalizations, has established connections to various domains of mathematics and physics, like intersection theory of the moduli space of curves and integrability. We will explain some of these properties and connections, giving several ideas why this is worth considering, and is the starting or gluing point of an active field of research, and finally hoping to instigate the search of new beautiful connections.

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• **Roland Speicher** (Saarbrücken Universität)
Free probability theory: higher orders and relation to topological recursion

Usual free probability theory was introduced by Voiculescu in the context of operator algebras. It turned out that there exists also a relation to random matrices, namely it describes the leading order of expectation values of the trace for multi-matrix models. Higher order versions of free probability were later introduced by Collins, Mingo, Śniady, Speicher in order to capture in the same way the leading order of correlations of several traces. A prominent role in free probability theory is played by "free cumulants" and "moment-cumulant formulas", and the underlying combinatorial objects are "non-crossing partitions" and, for the higher order versions, 'partitioned permutations".

I will give in my talks an introduction to free probability theory, with special emphasis on the higher order versions, and an eye towards possible relations to topological recursion. In particular, it seems that the problem of symplectic invariance in topological recursion has, at least in the planar sector, something to do with the transition between moments and free cumulants.

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• **Walter van Suijlekom** (Radboud University, Nijmegen)
Non-commutative geometry and spectral triples

Our starting point is a spectral approach to geometry, starting with the simple question 'can one hear the shape of a drum'. This was phrased by Mark Kac in the 1960s, and led to many developments in spectral geometry. For us, it is the motivation for considering spectral triples, which is the key technical device used to describe non-commutative Riemannian spin manifolds. We will give many motivating examples, and also explain how gauge symmetries naturally arise in this context.

The connection to the other main theme of the workshop is found via the spectral action principle. It allows for a derivation of an action functional from any given spectral triple. This includes the Hermitian matrix model, but more interesting matrix models appear beyond. We will consider some recent developments for such models by deriving a perturbative series expansion for the spectral action.

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Research talks

• **Séverin Charbonnier** (IRIF, Paris 7)
Topological recursion for fully simple maps

Fully simple maps show strong relations with symplectic invariance of topological recursion and free probabilities. While ordinary maps satisfy topological recursion for a certain spectral curve (x, y), G. Borot and E. Garcia-Failde conjectured in 2017 that fully simple maps satisfy topological recursion for the exchanged spectral curve (y, x). Two proofs of this conjecture were simultaneously released this year (`math-ph/2106.08368` and `math.CO/2106.09002`). I will first define fully simple maps and describe the context of the conjecture; second, I will detail the proof we provided together with G. Borot and E. Garcia-Failde via ciliated maps; last, some connections between this work and free probabilities will be drawn, along with questions left opened.

In collaboration with G. Borot and E. Garcia-Failde (`math.CO/2106.09002`).

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• **Kurusch Ebrahimi-Fard** (NTNU Oslo)
An operadic derivation of the twisted factorisation for the operator-valued T-transform

Together with Nicolas Gilliers, we have tried to understand how an operadic perspective might help to formulate a more transparent, i.e., combinatorial derivation of Dykema's twisted factorisation formula for the operator-valued T-transform. In this talk, we will discuss our approach using a diagrammatic presentation.

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• **Lisa Glaser** (University of Vienna)
A picture of a spectral triple

A compact manifold can be described through a spectral triple, consisting of a Hilbert space $\mathcal{H}$, an algebra of functions $\mathcal{A}$ and a Dirac operator $\mathcal{D}$. But what if we are given a spectral triple? Then the situation is more complicated, it is not clear how to reconstruct geometry from a spectral triple, in particular one with a non-commutative algebra $\mathcal{A}$, or a finite Hilbert space $\mathcal{H}$. But these are questions one would like to ask if trying to use spectral triples to possibly quantize gravity. In this talk I will show how we recover metric information from a truncation of a spectral triple to make an image, and show some odd shrinking spectral triples.

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• **Alexander Hock** (WWU Münster)
From noncommutative quantum field theory to blobbed topological recursion

Scalar quantum field theory on noncommutative Moyal space can be approximated by matrix models with non-trivial covariance. One example is the Kontsevich model, which is governed by topological recursion. We will focus on a slightly different matrix model with the same covariance as the Kontsevich model, but with a quartic interaction, the so-called quartic Kontsevich model. This model does not fit into the class of generalised Kontsevich models (which is also known to satisfy topological recursion). Our loop equations turn out to be of similar structure as the loop equations of the Hermitian 2-matrix model. However, computing the simplest expectation

values shows that this model is governed by blobbed topological recursion, an extension of topological recursion. This is proven for genus $g = 0$ and work in progress for higher genera. We will show the explicit recursive structure for these blobs by providing the explicit linear and quadratic loop equations (for $g = 0$).

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• **Roberta Iseppi** (Syddansk Universitet, Odense)
The BV-BRST cohomology for U(n)*-gauge theories induced by finite spectral triples*
The Batalin–Vilkovisky (BV) formalism provides a cohomological approach for the study of gauge symmetries: given a gauge theory, by introducing extra (non-existing) fields, we can associate to it two cohomology complexes, the BV and the BRST complex. The relevance of these complexes lies in the fact that their cohomology groups capture interesting physical information on the initial theory. In this talk we describe how both these complexes can be seen as Hochschild complexes of a graded algebra B over a bimodule M. By focusing on U(n)-gauge theories induced by a finite spectral triple on $M_n(\mathbb{C})$, we explain how the pair (B, M) is naturally encoded, respectively, in the BV spectral triple associated to the theory for the BV complex and in its gauge - fixed version for the BRST one. This result further reinforces the idea that the BV construction naturally inserts in the framework provided by noncommutative geometry.

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• **Camille Male** (Université de Bordeaux)
Freeness over the diagonal and the global fluctuations of Wigner matrices
We characterize the limiting second order distributions of independent complex Wigner and deterministic matrices using Voiculescu's notions of freeness over the diagonal. For unitary invariant random matrices, Mingo and Speicher's notion of second order freeness gives a universal rule to compute the global fluctuation. But this one is in general not valid for non Gaussian Wigner matrices, since the fluctuations are not universal, depending in particular on the moment of order 4 of the matrices. Yet, it is possible to adapt Mingo–Speicher's formulation and reformulate this notion for operator-valued random variables in a second order probability space, and prove a universal rule for more general Wigner matrices (for which the marginal second order distributions are not universal).

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• **James Mingo** (Queen's University, Kingston)
The infinitesimal Weingarten calculus
The Weingarten calculus calculates matrix integrals over the unitary and orthogonal groups, in particular their large N behaviour. In this talk we shall look at the Weingarten function on the orthogonal group and the term of subleading order and its relation to infinitesimal freeness. The leading order term was found by Collins and Śniady and governs asymptotic freeness. The subleading term was evaluated by Féray using complete symmetric functions. We will give another interpretation using the planar diagrams of infinitesimal freeness.

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• **Sergey Shadrin** (Universiteit van Amsterdam)
Arnold's trinity of algebraic 2d gravitation theories

"Arnold's trinities" refers to a metamathematical observation of Vladimir Arnold that many interesting mathematical concepts and theories occur in triples, with some hidden influence of R/C/H hidden in the background. By algebraic $2d$ gravitation theory I mean a very rich system of interrelated algebraic structures surrounding the concept of cohomological field theory in genus 0. It appears that there is an Arnold trinity of algebraic $2d$ gravitation theories (and one of them is a very natural non-commutative version of a CohFT), and I'll try to expose them, with a special focus on new homotopy quotients statements that we discovered last year.

A joint work with Vladimir Dotsenko and Pedro Tamaroff.

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• **Piotr Śniady** (Institute of Mathematics, Polish Academy of Science, Warsaw)
Representation theory from the random matrix perspective

In many cases a representation of a group can be viewed as a"random matrix with non-commutative entries". This viewpoint gives a heuristic explanation for many links between the representation theory, the random matrix theory and the free probability theory. This talk is intended to be easy and available to an audience which has no background in representation theory.

Supplementary material (slides, lecture notes, literature) will be available a couple of days before the lecture on the website `psniady.impan.pl/munster`

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• **Yan Soibelman** (Kansas State University)
Wall-crossing structures and exponential integrals

The notion of wall-crossing structure was introduced in my joint papers with Maxim Kontsevich for the purposes of Donaldson-Thomas theory (`math.AG/0811.2435`) and complex integrable systems (`math.AG/1303.3253`). In this talk I am going discuss a special case of this notion which appears in the theory of exponential integrals, including a multivalued case (i.e. exponential integral of a holomorphic one-form). This approach gives a conceptual explanation of the resurgence properties of arising perturbative expansions from the point of view of our general conjecture on analytic wall-crossing structures (`math.AG/2005.10651`). It also gives an alternative approach to the problem of counting of saddle connections on a Riemann surface.

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Gong talks (Monday 5pm)

• **Adam Afandi** (WWU Münster)
Hyperelliptic Hodge integrals

• **Nezhla Aghaei** (Syddansk Universitet, Odense)
Combinatorial quantisation of Chern Simons theory

• **Jan Boschheidgen** (Universidad Autónoma de Madrid)
Spectral measures associated to groups

• **Johannes Branahl** (WWU Münster)
Blobbed topological recursion meets quantum field theory

• **Nitin Kumar Chidambaram** (MPIM Bonn)
Topological recursion and Gaiotto vectors

• **Alessandro Giacchetto** (MPIM Bonn)
The Harer–Zagier formula via intersection theory

• **Marvin Hahn** (MPI Leipzig)
Bi-pruned Hurwitz numbers

• **Adrian de Jesus Celestino Rodriguez** (NTNU Oslo)
Semi-multiplicative functions and relations between cumulants

• **Reinier Kramer** (MPIM Bonn)
KP for Hurwitz-type cohomological field theories

• **Danilo Lewański** (IPhT CEA Saclay & IHES)
Topological Renaissance

• **Carlos Perez** (Institute for Theoretical Physics, Warsaw)
On multi-matrix models motivated by random noncommutative geometry

• **Giacomo Umer** (HU Berlin)
Topological recursion in the framework of Whittaker vectors