C*-Algebras: Tensor Products, Approximation & Classification (Münster 2023)

TITLES & ABSTRACTS

Hiroshi Ando, Chiba University.
Title: Some observations around Kirchberg's works on central sequence algebras.

Abstract: In this talk I will discuss some applications of Kirchberg's works on central sequence algebras, some of which are old and some new. In particular I report a work with Michal Doucha regarding the surjectivity of the homomorphism between approximately inner automoprhisms of unital C*-algebras induced by a quotient homomorphism between those C*-algebras. This can be seen as a consequence of Kirchberg's lemma on the central sequence algebra.

Bruce E Blackadar, University of Nevada, Reno.
Title: Hilbert Spaces Without Countable AC.

Abstract: We study Hilbert spaces, bounded operators, and C*-algebras without any form of the Axiom of Choice, particularly ones based on various types of Dedekind-finite sets. The situation is more varied and (arguably) more interesting in this setting, and there are some surprises. We obtain some new results in both operator algebras and set theory.

(Joint work with Ilijas Farah and Asaf Karagila)

Kristin Courtney, University of Münster.
Title: Nuclear C*-algebras as inductive limits of finite dimensional C*-algebras.

Abstract: Inductive limits are a central construction in operator algebras because they allow one to use well-understood building blocks to naturally construct more complicated objects whose properties remain tractable. In the classical setting, few nuclear C*-algebras arise as inductive limits of finite-dimensional C*-algebras. However, by generalizing our notion of an inductive system, Blackadar and Kirchberg were able to characterize quasidiagonal nuclear C*-algebras as those arising as (generalized) inductive limits of finite dimensional C*-algebras. In this talk, I will describe new notions of inductive systems which allow us to realize any nuclear C*-algebra as the limit of a system of finite dimensional C*-algebras. Though seemingly abstract, these systems correspond naturally to completely positive approximations of nuclear C*-algebras. This is based in part on joint work with Wilhelm Winter.

Joachim Cuntz, University of Münster.
Title: Eberhard as I have known him.

Abstract: I will try to give an impression of Eberhard Kirchberg as a mathematician and as a person.

Marius Dadarlat, Purdue University.
Title: A generalized Dixmier-Douady theory and applications.

Abstract: We give a succinct survey of the generalized Dixmier-Douady theory for bundles of stable strongly self-absorbing C*-algebras that we have developed in joint work with Pennig. Then, we present a computation of the cohomological invariants that classify these bundles, obtained in collaboration with McClure and Pennig. Finally, we discuss an application to amenable group actions on Kirchberg algebras based on results of Izumi, Matui, Meyer, Gabe and Szabo.

Mikael De La Salle, Université de Lyon 1
Title: A type III distributional inequality and application to quantum correlations.

Abstract: If two positive L^2 functions are close in the L^2 norm, then they admit sub-level sets that are also close in measure. This is a very easy exercise. In his 1976 work on the classification of factors, Alain Connes adapted this to matrices (and more generally tracial von Neumann algebras). This influencial inequality was recently used by Thomas Vidick to prove that synchronous games that admit good finite-dimensional strategies also admit good synchronous strategies, and this is relevant to the resolution of Kirchberg's QWEP conjecture (aka Connes' embedding problem). I will explain how to extend Connes' inequality to general von Neumann algebras, and how this allows to extend Vidick's reduction to infinite dimension. This is a joint work with Amine Marrakchi.

Søren Eiler, University of Copenhagen.
Title: Structure-preserving automorphisms of Cuntz algebras.

Abstract: Considering a Cuntz algebra $\mathcal{O}_n$ with $2 \leq n \leq \infty$ as the graph algebra associated to a directed graph with one vertex and n edges equips $\mathcal{O}_n$ naturally with a Cartan subalgebra and an action of the circle. Other graphs representing Cuntz algebras exist in abundance, and it is a pertinent question when such a graph defines the same C*-algebra in a way preserving Cartan subalgebras, circle actions, or both. One may usefully reformulate this as a question concerning automorphisms on a fixed C*-algebra.

Studying such problems for general graph algebras that are not necessarily simple or purely infinite has been a long undertaking in joint work with Ruiz. The simple case is much better understood, but there are still open problems. I will contrast our rather complete understanding for $\mathcal{O}_2$ with the several outstanding questions for $\mathcal{O}_\infty$, speaking almost exclusively about what I think we may safely assume were Eberhard's two favorite Kirchberg algebras.

George Elliott, University of Toronto.
Title: The scientific legacy of Eberhard Kirchberg—a preliminary look.

Abstract: I will attempt to isolate some of Eberhard's contributions, and what they have led to. (For instance, he proved his QWEP conjecture is equivalent to the Connes Embedding Problem, and it seems helps to understand that—many people have built on this, but I won't go into that.)

Ilijas Farah, York University.
Title: Some applications of property (T) groups.

Abstract: Discrete groups with property (T) and infinitely many irreducible representations on finite-dimensional Hilbert spaces continue to be a rich source of examples of operator algebras. I will talk about some recent progress.

James Gabe, University of Southern Denmark.
Title: Classification of purely infinite C*-algebras.

Abstract: As many of you know, Kirchberg had been working on a book on (non-simple) purely infinite C*-algebras and their classification theory. I will explain the main classification theorems (which I have given new proofs of a few years ago), and if time permits, I will explain some applications.

Priyanga Ganesan, University of California San Diego.
Title: Quantum Non-Local Games.

Abstract: In recent years, nonlocal games have received significant attention in operator algebras and resulted in highly fruitful interactions, including the recent resolution of the Connes Embedding Problem. A nonlocal game involves two non-communicating players (Alice and Bob) who cooperatively play to win against a referee. In this talk, I will provide an introduction to the theory of non-local games and quantum correlation classes. We will discuss the role of C*-algebras and operator systems in the study of their perfect strategies. It will be shown that mathematical structures arising from entanglement-assisted strategies for nonlocal games can be naturally interpreted and studied using tools from operator algebras. I will then present a general framework of non local games involving quantum inputs and classical outputs and use them to discuss a quantum graph coloring game.

Shirly Geffen, University of Münster.
Title: $\mathcal{Z}$-stability and dynamical comparison.

Abstract: $\mathcal{Z}$-stability of a crossed product $A \rtimes G$ for an outer action of an amenable group G on a classifiable C*-algebra A is conjectured to hold in general, and is established under certain dynamical assumptions. Extending the known cases, we prove, under certain freeness conditions on the induced action on the space of extreme traces of A, that $A \rtimes G$ is classifiable (modulo the UCT). Similar to recent classification results in the commutative setting, we use the new notion of dynamical comparison, in its noncommutative version (due to Bosa-Perera-Zacharias-Wu). This is joint work with Eusebio Gardella, Petr Naryshkin, and Andrea Vaccaro.

Ilan Hirshberg, Ben-Gurion University of the Negv
Title: Dimension theory for non-free group actions.

Abstract: I will discuss a few dimension-type invariants for discrete group actions on locally compact Hausdorff spaces, which are not assumed to be free, the most significant of which is something we call the long and thin cover dimension, motivated by work of Bartels, Luck and Reich. Those are used to show that the associated crossed product has finite nuclear dimension. The result applies to arbitrary actions of finitely generated virtually nilpotent groups on finite dimensional spaces, but also covers other classes of actions, such as hyperbolic groups acting on boundaries

of hyperbolic complexes (e.g. the Gromov boundary), as well as certain allosteric actions of wreath products of finite abelian groups by $\mathbb{Z}^d$. This generalizes and puts in a more conceptual framework previous work of ours on non-free actions of the integers. This is joint work in preparation with Jianchao Wu.

Nadia Slavila Larsen, University of Oslo.
Title: Higher rank graphs from cube complexes and their C*-algebras.

Abstract: We use a combinatorial procedure first introduced by Hazlewood-Raeburn-Sims-Webster to construct an infinite family of k-graphs from complexes with universal cover equal to a product of k regular trees, for all k greater than two. The resulting k-graphs are aperiodic in the sense of Kumjian-Pask and Sims, and rigid in the sense of Lawson-Vdovina, ensuring that their C*-algebras are purely infinite and simple. This is joint work with Alina Vdovina (CUNY).

Ralf Meyer, Georg-August-Universität Göttingen.
Title: Classifying actions of groups on Kirchberg algebras.

Abstract: The classification theorem for simple nuclear purely infinite C*-algebras in the bootstrap class has two aspects. The analytic aspect is due to Kirchberg, who showed that a KK-equivalence between suitable C*-algebras lifts to an isomorphism. The topological aspect shows that an isomorphism on K-theory lifts to a KK-equivalence. Equivariant variants of Kirchberg's theorem show that an equivalence between two objects in suitable equivariant KK-theories lifts to an equivariant isomorphism of Morita equivalence. For group actions, this equivalence usually amounts to a cocycle conjugacy. I will discuss some of my results on the topological aspects of this classification theorem. Using homological algebra in triangulated categories such as equivariant KK-theory, we may produce Universal Coefficient Theorems in certain situations, which then imply classification results. I will, in particular, discuss actions of finite groups of prime order, where a UCT by Manuel Köhler allows to classify actions in the equivariant bootstrap class up to equivariant KK-equivalence by a certain purely algebraic invariant.

Magdalena Elena Musat, University of Copenhagen.
Title: The Connes-Kirchberg Problem, quantum correlations and infinite dimensional phenomena in QIT.

Abstract: I will survey the recent developments in the analysis of quantum correlations and their deep interconnections with the multi-faceted Connes-Kirchberg Problem, leading also to infinite dimensional phenomena in QIT.

Narutaka Ozawa, Kyoto university.
Title: On skew amenability for the unitary groups of some nuclear C*-algebras.

Abstract: We prove that the unitary group of a unital $\mathcal{Z}$-stable nuclear C*-algebra with the QTS property (every nontrivial quotient has a nonzero tracial state) is skew amenable in the weak topology. This answers a problem posed by Alekseev, Schmidt, and Thom.

N. Christopher Phillips, University of Oregon.
Title: Large finite values of Rokhlin dimension with commuting towers.
(This is joint work with Ilan Hirshberg.)

Abstract: Let G be a compact Lie group, not necessarily connected (so that finite groups are included). Recall that an action of G on a compact space X is free if every nontrivial group element acts with no fixed points. Finite Rokhlin dimension (with commuting towers, the only version we consider here) is one of many possible noncommutative versions of freeness. To put it in context, the action of G on $G \times X$ by translation in the first coordinate has Rokhlin dimension zero, while the action $x \mapsto -x$ of $\mathbb{Z}_2$ on the sphere S^d has Rokhlin dimension d. For each G and each positive integer d, there is a "universal G-space" for actions with Rokhlin dimension d, and it was more or less already known to topologists that this space has Rokhlin dimension exactly d.

What are the possible values of Rokhlin dimension for actions on simple C*-algebras? Rokhlin dimension zero is just the Rokhlin property. Several examples are known with Rokhlin dimension exactly one, and one with Rokhlin dimension exactly two, but until now no examples were known to have Rokhlin dimension finite but greater than two, even without knowing the exact value.

We construct actions of finite groups and of connected compact Lie groups on simple C*-algebras, in some cases even simple AF algebras, for which we can prove that the Rokhlin dimension is large but finite. In some cases we can say more. For example, for every d, there is an action of the circle on a simple unital AH algebra whose Rokhlin dimension is exactly d. However, much remains open about the possible values of Rokhlin dimension for actions on simple C*-algebras.

Mikael Rørdam, University of Copenhagen.
Title: Around traces and quasitraces.

Abstract: It is a fundamental problem to decide if a (unital) C*-algebra admits a trace or a (2-)quasi-trace. While we do know a lot about this, our understanding of this problem is not complete. Most famously, we still don't know if all quasitraces are traces on non-exact C*-algebras. In this talk I will give a survey on selected highlights related to this problem, with emphasis on the work of Kirchberg. I will discuss Haagerup's proof that quasi-traces on exact unital C*-algebras are traces (in the version of Haagerup-Thorbjørsen) – a result extended by Kirchberg to the non-unital case. I will mention an example of an ultra-power of a sequence of simple unital C*-algebras neither of which admit a quasi-trace, but where the ultra-power does. This relates to a theorem by Ozawa describing traces on ultra-powers of C*-algebras, a result recently extended by Antoine-Perera-Robert-Thiel. If time permits, I will also talk about when a C*-algebra admits a faithful tracial state, and why this is interesting. Parts of the content of this talk are based on joint work with my former PhD student Henning Milhøj.

Christopher Schafhauser, University of Nebraska–Lincoln.
Title: Lifting *-homomorphisms.

Abstract: The systematic study of lifting *-homomorphisms from a quotient to a C*-algebra goes back to Brown-Douglas-Filmore theory, where they consider the problem of lifting a *-homomorphism from a commutative C*-algebra into the

Calkin algebra to a *-homomorphisms into the algebra of bounded operators. They prove such a lift exists if and only if a homological obstruction in Ext vanishes, and using a preliminary version of the UCT, they further relate this obstruction to index theory. I will discuss a more general lifting problem, where under suitable regularity conditions, a lift will exist precisely when KK-theoretic obstructions vanish. Particular focus will be given to lifting problems related to the classification of non-unital simple nuclear C*-algebras. This is based on joint work with Jose Carrion, Jamie Gabe, Aaron Tikuisis, and Stuart White.

Karen Strung, Czech Academy of Sciences, Prague.
Title: Realising quantum flag manifolds as graph C*-algebras.

Abstract: In this talk I will show how the C*-completions of the so-called quantum flag manifolds—noncommutative spaces arising as homogeneous spaces of quantum groups—can be realised as graph C*-algebras. After recalling the definition of a quantum flag manifold and its C*-algebra, I will describe how to compute the primitive ideal space using Dijkhuizen and Stokmann's description of a complete set of irreducible *-representations. This allows one to to construct a graph directly from the Weyl group of the associated Lie algebra, and appeal to classification results of Eilers, Ruiz and Sorensen to show that this graph C*-algebra is isomorphic to the C*-algebra of the relevant quantum flag manifold. This recovers some known isomorphisms between the C*-algebras of quantum flag manifolds, as well as determining surprising new ones.

Joint work with Tomasz Brzeziński, Ulrich Krähmer, and Réamonn Ó Buachalla.

Gabor Szabo, KU Leuven.
Title: The dynamical Kirchberg-Phillips theorem.

Abstract: In this talk I will present the main results of a joint work with James Gabe. Given a countable discrete group G, two amenable and outer G-actions on stable Kirchberg algebras are cocycle conjugate precisely when they are equivariantly KK-equivalent. This re-proves and generalizes the celebrated classification theorem by Kirchberg-Phillips. In fact this result for discrete groups arises as the special case of a theorem covering the case where G can be any second-countable locally compact group, albeit with more delicate assumptions in that generality. After discussing the core concepts playing a role in our theory, I will state the classification results. If time permits, I will comment on some interesting consequences.

Simon Wassermann, University of Glasgow.
Title: Characterizations of C*-exactness.

Exactness of a C*-algebra A was originally defined in terms of a flatness condition, that the operation of taking the minimal tensor product $A\otimes\_$ by A preserve exactness of a short exact sequences of C*-algebras. Kirchberg showed that A is exact if and only if it has a faithful representation on a Hilbert space with the representation map nuclear and from this was able to show that any separable exact C*algebra A is a subquotient of the CAR algebra B, that is, $A \cong G/J$ for some C*-subalgebra G of B and ideal J of G. Using conditions C and C$'$ introduced by Archbold and Batty in a 1980 paper, he was able to show that the converse holds: any subquotient

of the CAR algebra is an exact C*-algebra. He subsequently strengthened this result by showing that any separable exact C*-algebra can be embedded in a nuclear C*-algebra. In fact he found several very different proofs, one in collaboration with Phillips, that a separable exact C*-algebra is $*$-isomorphic to a C*-subalgebra of the Cuntz algebra $\mathcal{O}_2$.

In a piece of unpublished work, Kirchberg gave a direct proof, with no separability assumptions, that properties C and C$'$ are equivalent to exactness. This result and its relevance to the problem of embedding an inseparable exact C*-algebra in a nuclear C*-algebra will be considered in this talk.

Stuart White, Oxford University.
Title: $\mathcal{Z}$-stability and KK-uniqueness.

Abstract: KK-uniqueness asks when a Cuntz pair $(\phi, \psi)\colon A \rightrightarrows \mathcal{M}(I) \rhd I$ of absorbing representations which is zero in $KK(A, I)$ are properly asymptotically unitarily equivalent. I'll discuss this problem (defining all the terminology) and how a partial solution in the presence of $\mathcal{Z}$-stability plays a role in classification. This is an extract from joint work with Carrion, Gabe, Schafhauser and Tikuisis