Universität Münster

Arithmetic Algebraic Geometry

A conference on the occasion
of Michael Rapoport's 75th birthday

October 2 – 6, 2023
Münster, Germany

Organizers

Eugen Hellmann (Münster)
Andreas Mihatsch (Bonn)
Timo Richarz (Darmstadt)
Eva Viehmann (Münster)

living.knowledge

Schedule
Arithmetic Algebraic Geometry
October 2 - October 6, 2023

Monday	
08:00 - 9:00	Registration
9:00 - 10:00	Howard
10:00 -10:30	Coffee
10:30 - 11:30	Pilloni
11:30 -13:30	Lunch
13:30 -14:30	Vignéras
14:30 -15:00	Coffee
15:00 - 16:00	Zhou
16:00 -16:20	Short break
16:20 - 17:20	Xu

Tuesday	
9:00 - 10:00	Zhiyu Zhang
10:00- 10:30	Coffee
10:30 -11:30	Li
11:30 -13:30	Lunch
13:30 -14:30	Chen
14:30 - 15:00	Coffee
15:00 - 16:00	Fargues

Wednesday	
9:00 - 10:00	Mézard
10:00 - 10:30	Coffee
10:30 - 11:30	Levin
11:30 - 11:45	Short break
11:45 - 12:45	Emerton
	Lunch / Free afternoon/ City tour
18:30	Dinner

Thursday	
9:00 - 10:00	Wei Zhang
10:00 - 10:30	Coffee
10:30 - 11:30	Feng
11:30 - 13:30	Lunch
13:30 - 14:30	Lourenço
14:30 - 15:00	Coffee
15:00 - 16:00	Hansen
16:00 - 16:20	Short break
16:20 - 17:20	Scholze

Friday	
9:00 - 10:00	Niziol
10:00 - 10:30	Coffee
10:30 - 11:30	Görtz
11:30 - 13:30	Lunch
13:30 - 14:30	Kisin

General information

Registration:

Registration takes place in the building Einsteinstr. 62, 48149 Münster. On the ground floor in room SR 0 (see map on the next page).

Lectures:

The lectures will take place in M2 in the lecture hall building, Einsteinstr. 64.

Coffee break:

The coffee breaks will take place in the Seminarraumzentrum (SRZ) at Orléansring 12 on the 2nd floor.

Wi-Fi access:

If you are part of the eduroam community, you may connect to the network "eduroam" as usual. Otherwise you can connect to the SSID "GuestOnCampus" and start any web browser. You will automatically be redirected to the login page. Confirm the terms of use and click on "log in for free". 1 GB data volume is available per device and day. Please note that the connection is not encrypted.

Bank holiday:

Tuesday, 3 October, is a public holiday in Germany.
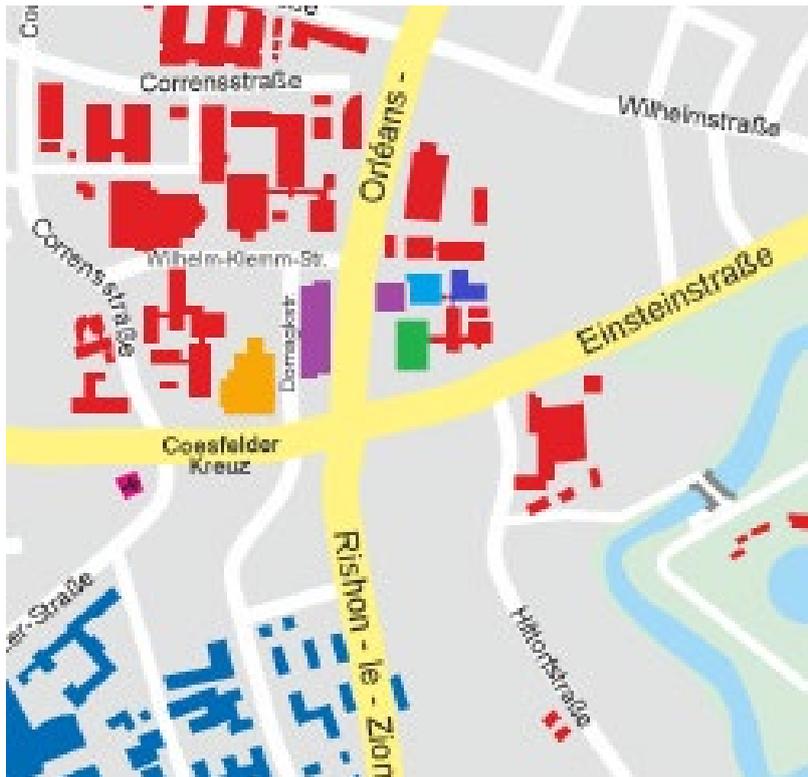
Lecture hall building (Talks will take place in M2.)
SRZ (Coffee breaks on the 2nd floor.)
MM building
Canteen
Parking lot

Lunch:

There are a couple of restaurants for lunch in the vicinity:

- Canteen – Mensa am Ring, Domagkstraße 61 (most convenient option, even if not the most idyllic place. Cash payment only.) (closed on Tuesday)

- Ristorante Milano (Italian), Wilhelmstraße 26 (closed Mondays)

- Il Gondoliere (Italian), Von-Esmarch-Straße 28 (closed Mondays)

- Buddha Palace (Indian), Von-Esmarch-Straße 18 (closed Tuesdays)

- La Gondola D'oro (Italian), Hüfferstraße 34

- Gustav Grün (Green Fast Food), Wilhelmstraße 1

- Áro (Green Fast Food), Neutor 3

Public transportation

You can check the bus schedule on the web-site of [Stadtwerke-Münster](#) (in German and English), or use Google maps.

City tour on Wednesday afternoon

The city tour takes place at 16:00. Registration is needed.

All participants of the city tour have already received an email with information about the location, time and group allocation.

Free afternoon on Wednesday

Suggestions and ideas for sightseeing in Münster:

Visit the castle, the park behind it and the embedded botanical garden. Visit a museum, e.g. the LWL Museum of Art and Cultural History or the Picasso Museum. Take a walk around the Aasee or familiarise yourself with European history at the Historic Town Hall, one of the two places where the Peace of Westphalia was signed in 1648.

Conference dinner on Wednesday

The conference dinner takes place on Wednesday at 18:30 at the Restaurant "Schlossgarten" (Schlossgarten 4, 48149 Münster).

Acknowledgments

Book of abstracts

Miaofen Chen (Shanghai)   *Tue 13:30*

A Harder-Narasimhan stratification in p-adic Hodge theory

We consider the Harder-Narasimhan formalism on the category of normed isocrystals and show that the Harder-Narasimhan filtration is compatible with tensor products which generalizes the result of Cornut for normed vector spaces. As an application of this result, we are able to define a Harder-Narasimhan stratification on the B_{dR}^+-Grassmannian for arbitrary (G, b, μ). When μ is minusucle, it corresponds to the Harder-Narasimhan stratification on the flag varieties defined by Dat-Orlik-Rapoport. While b is basic, it's studied by Nguyen-Viehmann and Shen. We study the basic geometric properties of the Harder-Narasimhan stratification, such as non-emptiness, dimension and its relation with other stratifications. This is a joint work in progress with Jilong Tong.

Matt Emerton (Chicago)   *Wed 11:45*

tba

Laurent Fargues (Paris)   *Tue 15:00*

Laumon sheaf and the mod p Langlands program for GL_2 of a finite degree extension of Qp

Let E be a finite degree extension of Qp. Given a mod p representation of the absolute Galois group of E we construct a sheaf on a punctured absolute Banach-Colmez space that should give the first step in the construction of the mod p local Langlands correspondence as a representation of the mirabolic subgroup of $GL_2(E)$. We will explain the construction of this sheaf, its conjectural properties and the link with the recent work of Breuil, Herzig, Hu Morra, and Schraen about local-global compatibility.

Ulrich Goertz (Duisburg-Essen)   *Fri 10:30*

Quiver Grassmannians and local models

Given a representation of a quiver and a dimension vector, the associated quiver Grassmannian is the scheme of subrepresentations of the given dimension vector.
It is known that every projective scheme over a field is isomorphic to a quiver Grassmannian for a ``suitably complicated'' quiver.

On the other hand, restricting the choice of quiver and representation one obtains well-behaved schemes. Quiver Grassmannians arise naturally in connection with moduli spaces of different kinds. For example, the Rapoport-Zink local model for the general linear group is an example of a quiver Grassmannian with particularly favorable properties. Another example is the linked Grassmannian introduced by Osserman.

In the talk, I want to explain some connections between the geometry of these spaces and the representation theory of quivers, and in particular discuss when a quiver Grassmannian can be described by ``simple'' matrix equations.

David Hansen (Bonn)   *Thu 15:00*

Local Shimura varieties - past, present, future

Local Shimura varieties, roughly speaking, are moduli spaces of p-adic Hodge structures. Aside from having a fascinating and complicated geometry, their cohomology is intimately connected with the Langlands correspondence. I will give an introduction to the history of this circle of ideas, discuss some exciting recent developments, and highlight some intriguing open problems.

This talk is at the same time a Colloquium talk at the Institute of Mathematics.

Benjamin Howard (Boston College)   *Mon 9:00*

Supersingular loci of GU(2,n-2) Shimura varieties

Rapoport-Zink spaces are defined as moduli spaces of p-divisible groups with additional structure. Although general existence theorems are known for these spaces, there are relatively few cases in which one understands their structure in any kind of explicit way. In this talk, I will give a concrete and reasonably explicit description of (the reduced schemes underlying) Rapoport-Zink formal schemes of type GU(2,n-2). This is joint work with Mia Fox and Naoki Imai.

Mark Kisin (Harvard)   *Fri 13:30*

Heights in the isogeny class of an abelian variety

Let A be an abelian variety over an algebraic closure of Q. A conjecture of Mocz asserts that there are only finitely many isomorphism classes of abelian varieties isogenous to A, and of height less than some fixed constant c.
In this talk, I will sketch a proof of the conjecture when the Mumford-Tate conjecture - which is known in many cases - holds for A. This result should be compared with Faltings' famous theorem, which is about finiteness for abelian varieties defined over a fixed number field. This is joint work with Lucia Mocz.

Brandon Levin (Arizona)   *Wed 10:30*

Cycles on the Emerton-Gee stack

The Breuil-Mezard conjecture predicts the geometry of local deformation rings with p-adic Hodge theory conditions in terms of modular representation theory. I will describe a version of this conjecture on the Emerton-Gee moduli stack of mod p Galois representations and its connection with the weight part of Serre's conjecture. I will then overview progress towards these conjectures and related problems.

Chao Li (Columbia)   *Tue 10:30*

Kudla-Rapoport conjecture for Krämer models

The Kudla-Rapoport conjecture, proved jointly with Wei Zhang, is a precise identity relating arithmetic intersection numbers of special cycles on unitary Shimura varieties with good reduction and central derivatives of Siegel Eisenstein series. We discuss how to formulate and prove an analogous identity for certain unitary Shimura varieties with bad reduction (Krämer models at ramified places). We will motivate these conjectures, mention applications to L-functions and highlight interesting new phenomena in the presence of bad reduction. This is joint work with Qiao He, Yousheng Shi and Tonghai Yang.

João Lourenço (Münster)   *Thu 13:30*

Local models revisited

Local models were first systematically introduced by Rapoport--Zink in terms of linear algebra to study the local geometry of RZ spaces and Shimura varieties. It was observed by Görtz that they naturally embed into affine Grassmannians, so several tools from geometric Langlands were used along the years to prove quite strong results on their singularities, most notably by Faltings, Pappas, Rapoport, and Zhu. Recently, advances in p-adic geometry by Scholze and Zhu allow to consider local models over p-adic fields in a more functorial way and even beyond the minuscule case. We will give an overview of our latest contributions to these problems, based on joint works with a subset of the following collaborators: Anschütz, Cass, Fakhruddin, Gleason, Haines, Richarz.

Ariane Mézard (Paris)   *Wed 9:00*

Local model theory for non-generic tame potentially Barsotti-Tate deformation ring

In collaboration with B. Le Hung and S. Morra, we develop a local model theory for moduli stacks of étale (φ, Γ)-modules corresponding to 2-dimensional non-scalar tame potentially Barsotti-Tate Galois representations of the Galois group of an unramified extension of $\mathbf{Q}_p$.

We derive from this explicit presentations of potentially Barsotti-Tate deformation rings, in particular the proof of the various Caruso-David-Mézard conjectures.

Wieslawa Niziol (Paris)   *Fri 9:00*

Duality for p-adic pro-etale cohomology of Stein spaces

I will discuss duality theorems in p-adic pro-étale cohomology of Stein spaces. This is based on a joint work with Pierre Colmez and Sally Gilles.

Vincent Pilloni (Paris)   *Mon 10:30*

On the modularity of abelian surfaces

Following work of Pan and Rodriguez we develop a p-adic Eichler-Shimura theory for completed cohomology. We give an arithmetic application to abelian surfaces.

This is joint work with G. Boxer, F. Calegari and T. Gee.

Peter Scholze (Bonn)   *Thu 16:20*

Towards a formulation of the real local Langlands correspondence as geometric Langlands on the twistor-$\mathbf{P}^1$

We will propose a formulation of the local Langlands correspondence for complex representations of real groups in terms of a(n everywhere unramified) geometric Langlands correspondence on the twistor-$\mathbf{P}^1$, analogous to our work with Fargues in the case of p-adic groups. This is motivated by discussions with Rodriguez Camargo, Pan, le Bras and Anschütz on the analogous case of locally analytic p-adic representations, and is different from the previous work of Ben-Zvi and Nadler in a similar direction. In particular, on the geometric side we get representations of the real group, encoded in terms of liquid quasicoherent sheaves on $[*/G(\mathbf{R})^{la}]$; and on the spectral side, we get representations of the real Weil group $W_{\mathbf{R}}$, or rather vector bundles on $[(\mathbf{A}^2\backslash\{0\})/W_{\mathbf{R}}^{la}]$.

Marie-France Vignéras (Paris)   *Mon 13:30*

Restriction of admissible representations to small Moy-Prasad subgroups

Let F be a finite extension of $\mathbf{Q}_p$ or $\mathbf{F}_p((t))$, G the group of rational points of a reductive F-group, R a field. For an irreducible admissible R-representation π of G, the asymptotic dimension of the invariants of π by the congruence subgroups of a Moy-Prasad subgroup of G is probably polynomial (in progress with Guy Henniart).

Yujie Xu (MIT)   *Mon 16:20*

Uniformizing the moduli stacks of global G-shtukas and applications

Moduli stacks of global G-Shtukas play an important role in the global Langlands program over function fields. They were used in the proof of the function field global Langlands conjecture by Drinfeld, Lafforgue etc. In this talk, I will speak on my joint work with Urs Hartl, where we show that the moduli spaces of suitably bounded global G-Shtukas with colliding legs satisfy a p-adic uniformization isomorphism by Rapoport-Zink spaces. If time permits, I will mention some applications (e.g. to the Langlands-Rapoport conjecture over function field).

Zhiyu Zhang (MIT)   *Tue 9:00*

Mirabolic special cycles and the twisted AFL

Kudla-Rapoport cycles on unitary Rapoport-Zink spaces and Shimura varieties realize arithmetic theta liftings for hermitian spaces, and are related to central derivatives of automorphic generating functions. In this talk, we will introduce and study mirabolic special cycles on certain Rapoport-Zink spaces for GL_n, which do not exist in the "real" world. They satisfy a symmetry under duality, recover Kudla-Rapoport cycles via pullbacks, and could be used to reformulate (more) Kudla-Rapoport conjectures. Moreover, we will formulate a twisted arithmetic fundamental lemma in the context of twisted arithmetic Gan—Gross—Prasad conjectures. With the help of these new cycles we can prove this new AFL.

Wei Zhang (MIT)   *Thu 9:00*

Tony Feng (Berkeley)   *Thu 10:30*

Generic modularity for Shtuka special cycles: (I) and (II)

Classical theta functions and their generalization to generating series of special cycles on Shimura varieties have been active themes for a long time. In this two-talk series, we aim to explore the new feather of their function field analog.

In the first talk (by Wei), we will present the definition of special cycles on the moduli stack of unitary Shtukas. We form a generating series on the quasi-split unitary group U(m,m) with coefficients in Chow groups, and conjecture their modularity. We present a recent theorem on the modularity upon taking the cycle classes in the cohomology of the generic fibers of the moduli stacks.

In the second talk (by Tony), we will indicate some ideas that go into the proof of the generic modularity theorem. We will highlight some new general tools that we introduced, which may have wider applicability. One is a "sheaf-cycle" correspondence that allows us to deploy sheaf-theoretic arguments in the analysis of algebraic cycles, in the style of the classical sheaf-function correspondence. Another is a theory of derived Fourier analysis, which expands the Deligne-Laumon Fourier transform to the setting of derived algebraic geometry.

This is a join work of the two speakers with Zhiwei Yun.

Rong Zhou (Cambridge) *Mon 15:00*

Strongly compatible systems associated to abelian varieties

Let A be an abelian variety over a number field E and r_l the representation of the absolute Galois group of E on its l-adic Tate module. Then a classical result of Weil, Deligne and Raynaud implies that $(r_l)_{l \text{ prime}}$ forms a strongly compatible system. Explicitly, this means that the Weil-Deligne representation associated to the restriction of r_l to the decomposition group at each finite place satisfies an l-independence property.

In this talk I will discuss recent and ongoing work with Kisin towards proving a motivic refinement of this result, namely that upon replacing E by a finite extension, the r_l can be refined to a strongly compatible system of G-valued representations, where G is the Mumford-Tate group of A.